Algorithmic Trading: Market Impact Models and Trade Scheduling

Ekaterina Kochieva

Gautam Mitra

Cormac A. Lucas

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Introduction

- Making the best financial decision, that is, achieving higher return and lower risk is the holy grail of financial planning.
- Portfolio choice and asset allocation is a strategic decision. Buying and selling of assets in a time bound schedule is a matter of tactics. The importance of the latter in respect of the former is rapidly rising in the consciousness of the financial community.
- Computerisation of major stock exchanges stimulates the growing popularity of trading algorithms. Speed and effectiveness of implementing an investment decision and the resulting trade have become key factors for success.

Contributing factors

- Price dynamics (price trend):
  - natural price movement of a stock
  - forecasting model (macroeconomic analysis , historical data analysis)
  - estimate price for trading horizon (constant price change, percentage return, exponential growth rate techniques)
- Market impact (price slippage):
  - the trade disturbs market equilibrium price
  - long-duration **permanent** component caused by information leakage
  - **temporary** price jump due to liquidity demand

Contributing factors

- Market impact models:
  - power law function of volume
    - linear
    - square root
  - weighted sum of linear and non-linear components
  - product of instantaneous impact function and a decay function

Contributing factors

- Timing risk corresponds to uncertainty of the order's true execution cost
  - price volatility - price is higher/lower than expected
  - liquidity risk – market volume is uncertain
  - estimation error of market impact parameters

Contributing factors

- Opportunity cost – lost profit of not being able to implement an investment decision in full
  - Insufficient liquidity
  - Extreme and fast unwanted price movement
- Investment-related component: a manager delays sending an order to a trader, a trader delays releasing the order to the market. This delay can be reduced
- Trading-related component – the actual cost related to inability of completing a trade order
- Opportunity cost is a function of (i) price movement and (ii) the quantity of unexecuted shares

Price benchmarks

- Choose a benchmark price to measure execution costs and trading effectiveness
- Implementation shortfall – difference between actual portfolio return and paper return
- Pre-trade – good proxy to compute delay and execution cost. Arrival price benchmark is popular
- Intraday – good indication of fair market price (e.g. VWAP)
- Post-trade – if combined with pre-trade benchmark gives an estimate of the opportunity cost of the order

Overview of some strategies

- VWAP strategy
- Efficient trading frontier
- Static strategy
- Dynamic strategy
- Adaptation tactics
- Stochastic programming approach

Implementation goal

- Traders dilemma:
  - Market impact is a decreasing function with time and volume.
  - Timing risk is an increasing function with time and volume.
  - So, trading too aggressively cause traders to incur high market impact cost and low timing risk. Trading too passively means having low market impact cost but high timing risk

Model objectives

- Minimise cost with specified level of risk
- Minimise risk for specified level of cost
- Minimise risk (risk/ tracking error/ value-at-risk) for a specified cost, based on a fair value calculation
- Balance trade-off between cost and risk via risk aversion parameter λ
- Use utility function (maximise)

VWAP strategy

- VWAP - Volume Weighted Average Price

$$VWAP = \frac{\sum_j v_j p_j}{\sum_j v_j}$$

- Trading a fixed percent of market volume in each period. It minimises market impact cost but not necessarily overall cost of execution. Often used when cost is measured against VWAP benchmark. Allows to hide the true size of the order.
- If the expected price appreciation cost is zero, VWAP strategy gives the least cost

Efficient trading frontier

$$\underset{\alpha}{Min} \quad Cost(\alpha) + \lambda \cdot Risk(\alpha)$$

- α – trading rate as a fraction of volume
- λ – risk aversion parameter

- Solving for various levels of λ give a set of trading strategies each with different cost and risk, which form the efficient trading frontier (introduced by Almgren & Chriss)

Static strategy

- Entire trade schedule is fixed in advance
- Parameters are estimated before the trade starts and do not change during execution of the strategy
  - Mean
  - Variance
- Predetermined risk aversion level λ

Dynamic strategy

- Arbitrary modification of the strategy at any time using the information on changing market conditions
- Trade list is recalculated using all available at the moment information
- Further strategy balances mean-variance trade-off of the remaining cost
- Risk aversion parameter λ is constant

Static or dynamic?

- Almgren &Chriss (2000):
  - price process – arithmetic random walk
  - no serial correlation
  - liquidity and volatility are known in advance
  - static strategy is equivalent to dynamic strategy for this model

Daily cycle

- During a trading day morning trading activity is different from afternoon. An intelligent trader collects information in the morning before he trades
- Most market models assume trade times are random and price behaviour is random
- Speed of trading may be adjusted according to changing market conditions
- Almgren & Lorenz (2006)
  - simple model which minimises costs taking in account momentum in price movement based on daily trading cycles
  - optimal risk-neutral adaptive strategy

Adaptation tactics

- Kissell & Malamut (2006)
  - Target cost
  - Aggressive-in-the-money (AIM)
  - Passive-in-the-money (PIM)
- In case of positive serial correlation if the price moves in one’s favour, he should trade more passively to capture even better price in the future. So PIM strategy is optimal
- If the price is believed to be mean-reverting, favourable price change is captured quickly

Dynamic strategy with fixed rule

- Almgren & Lorenz (2007)
- Negative correlation between gain/losses and following price impact
  - If the price moves favourably in the early part of trading, speed up the execution and spend the gains
  - If the price moves unfavourably – slow down and that will reduce future costs, despite the growing risk exposure
- Trade rate is a function of price and is determined using a mean-variance trade-off before trading begins. Arrival price measure is utilised
- The rule cannot be modified after the execution has started

Stochastic programming approach

- Krokhmal & Uryasev (2007)
- Problem of optimal position liquidation in presence of market impact
- Sample-path approach. Path-grouping to achieve non-anticipativity
- Some features:
  - Dynamic response to market conditions
  - Ability to use various market impact models
  - Incorporating historical data
  - Different types of constraints may be used
  - No restrictions on price process of a stock

Combined investment + implementation optimisation

- Engle & Ferstenberg (2006)
  - Mean-variance framework for both investment and execution
  - Risk aversion parameter is the same
- A single optimisation to discover the relation between investment and trading problems
- Hedging trading risk can make execution optimal

Summary

- The necessity of portfolio rebalancing makes an investor take trading risks. Thus, he requires a proper quantitative framework to manage transaction costs. Trade scheduling models can provide the appropriate order slicing scheme to improve portfolio returns during and after rebalancing. Market impact, price appreciation, timing and opportunity risks contribute towards overall trade cost and need to be taken into consideration to derive optimal trading decisions. There exist various trade algorithms and an investor can choose those that are consistent with his investment objectives.

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Thank you!